# John von Neumann e i suoi dubbi su rigore matematico e rappresentazione digitale

Teresa Numerico

tnumerico@uniroma3.it

Giornata seminariale Roma Tre, 9 luglio 2010

# John Von Neumann (1903-1957)



# outline

- Il rigore in matematica: un’analisi rispetto alle conseguenze dei problemi sui fondamenti
- Digitale vs  analogico rispetto a macchine e sistema nervoso
- La logica dei *self-reproducing automata*

# Il rigore in matematica

Un concetto liquido definito da storia, esperienza ed epistemologia

# Logica, matematica e storia

- Just as languages like Greek or Sanskrit are historical fact and not absolute logical necessities, it is only reasonable to assume that **logics and mathematics are similarly historical, accidental forms of expression**. They may have essential variants, i.e. they may exist in other forms than the ones to which we are accustomed. [...]

- We have now accumulated sufficient evidence to see that **whatever language the central nervous system is using**, it is characterized **by less logical and arithmetical depth** than what we are normally used to.

Von Neumann 1958 the computer and the brain: 81

# Il rigore matematico tra esperienza e filosofia

- This example [the foundations of arithmetic], however, deals with the relationship of mathematics with philosophy or epistemology rather than with the natural sciences. It illustrates in a very striking fashion that the very concept of "absolute" mathematical rigour is not immutable. The variability of the concept of rigour shows that something else besides mathematical abstraction must enter into the makeup of mathematics. In analyzing the controversy about the "foundations," I have not been able to convince myself that the verdict must be in favour of the empirical nature of this extra component. The case in favour of such an interpretation is quite strong, at least in some phases of the discussion. But I do not consider it absolutely cogent.

Von Neumann 1947

http://www-history.mcs.stand.ac.uk/Extras/Von_Neumann_Part_1.html

# Qualcosa di non matematico entra nella matematica...

- [It is clear] that something nonmathematical, somehow connected with the empirical sciences or with philosophy or both, does enter essentially-and its non-empirical character could only be maintained if one assumed that philosophy (or more specifically epistemology) can exist independently of experience.

Von Neumann 1947

# Digitale vs analogico

Nel sistema nervoso e nelle macchine

# Comprendere il cervello?

- Secondo von N. comprendere il cervello con gli studi neurologici sarebbe impossibile
- Senza contare che: the system is not even purely digital (i.e. neural): it is intimately connected to a very complex analogy (i.e. humoral or hormonal) system, and almost every feedback loop goes through both sector. If not through the “Outside” world (the world outside the epidermisor with the digestive system) as well.

Lettera di von Neumann a Wiener 29 nov. 1946

# Il funzionamento del sistema nervoso

- Yet the nervous system seems to be somewhat more flexibly designed [than the multiplexed automaton]. Also, its “digital” (neural) operations are rather freely alternating with “analog” (hormonal) processes in their complete chain of causation. Finally, the whole logical pattern of the nervous system seems to deviate in certain important traits qualitatively and significantly from our ordinary mathematics and mathematical-logical mode of operations

Von Neumann 1952 Probabilistic logics...: 87-88

# Com’è costituito il problema della conoscenza cerebrale

- Per comprendere il sistema nervoso è necessario innanzi tutto comprendere il funzionamento delle sue parti elementari e successivamente studiare come sono organizzate in un tutto funzionante
- Immaginando che la prima parte sia già ottenuta o dandola per scontata è possibile procedere attraverso una procedura di assiomatizzazione che possa rappresentare l’organizzazione di queste parti
- In questo caso le singole parti saranno trattate come black boxes dalle caratteristiche funzionali già note, interpretate come automatismi internamente non indagati che però al manifestarsi di certi stimoli non ambigui reagiranno attraverso delle risposte non ambigue

von N. 1948/1961, p. 289

# I problemi dell'assiomatizzazione

- I need not emphasize the limitations of this procedure. Investigations of this type may furnish evidence that the system of axioms used is convenient and, at least in its effects, similar to reality. They are, however, not the ideal method, and possibly not even a very effective method to determine the validity of the axioms. Such determinations of validity belong primarily to the first part of the problem.

Von Neumann 1948/1961: 290

# La comparazione tra calcolatori e organismi viventi...

- The computing machines, at least in their recent forms to which I am referring [...], are purely digital. Thus I must ask you to accept this oversimplification of the system. Although I am well aware of the analogy component in living organisms, and it would be absurd to deny its importance, I shall, nevertheless, for the sake of the simpler discussion, disregard that part.

Von Neumann 1948/1961: 297

# La logica degli automi

## Autoriproduzione e complessità

# La logica in termini di automi

- I risultati di Gödel  e Turing permettono di guardare alla logica in termini di automi
- Le proposizioni logiche possono essere rappresentate come reti elettriche o sistemi nervosi idealizzati
- Ma c'è una grande differenza: il ruolo del tempo che è centrale negli automi mentre è escluso dalla logica
- Il tempo implica la gestione delle risorse, la loro disponibilità e l'analisi sulla dimensione dei calcoli

# I problemi aperti dalla logica degli automi

- La distinzione tra digitale e analogico
- L’affidabilità di dispositivi costituiti da componenti instabili e la gestione degli errori
- Il ruolo della complessità
- La differenza tra automi naturali paralleli e automi artificiali seriali nelle operazioni

# Le debolezze della logica formale

- [Logic] deals with rigid, all-or-none concepts, and has very little contact with the continuous concept of the real or of the complex numbers, that is, with mathematical analysis. [...] Thus **formal logic is, by the nature of its approach, cut off from the best cultivated portions of mathematics**, and forced onto the most difficult part of the mathematical terrain, into combinatorics [...]

- In studying the functioning of automata, it is clearly **necessary to pay attention to a circumstance which has never before made its appearance in formal logic**

Von Neumann 1948/1961:303

# Gestire gli errori

Natural organisms are constructed to make errors as inconspicuous, as harmless, as possible. Artificial automata are designed to make errors as conspicuous, as disastrous as possible. [...] Natural organisms are sufficiently well conceived to be able to operate even when malfunctions have set in. [...]

Von Neumann 1948/1961: 306

# La logica dell'analogia

- Nobody would attempt to describe and define within a practical amount of space the general concept of analogy which dominates our interpretation of vision [...]
- We are dealing here with parts of logic with which we have practically no past experience. **The order of complexity is out of all proportion to anything we have ever known**
- We have no right to assume that the logical notations and procedures used in the past are suited to this part of the subject [...]
- It is therefore, not at all unlikely that it is futile to look for a precise logical concept, that is, for a precise verbal description of "visual analogy"

Von Neumann 1948/1961: 311

# La teoria della complicazione: ancora organizzazione

- [...] “complication” on its lower lever is probably degenerative, that is, that every automaton that can produce other automata will only be able to produce less complicated ones.
- There is, however, a certain minimum level where this degenerative characteristic ceases to be universal. At this point automata which can reproduce themselves, or even construct higher entities, become possible.
- This fact, that **complication**, as well as **organization**, **below a certain minimum level is degenerative, and beyond that level can become self-supporting and even increasing**, will clearly play an important role in any future theory of the subject

Von Neumann 1948/1961:318

# La logica degli automi

The logic of automata will differ from the present system of formal logic in two relevant respects:

1. The actual length of “chains of reasoning”, that is, of the chains of operations, will have to be considered.
2. The operations of logic [...] will have to be treated by procedures which allow exceptions (malfunctions) with low but non-zero probabilities. All of this will lead to theories which are much less rigidly of an all-or-none nature than past and present formal logic. They will be of a much less combinatorial, and much more analytical, character.

Von Neumann 1948/1961: 304

# Rigore assoluto e precisione

- It is perfectly true that in all mathematical problems the answer is required with absolute rigor, with absolute reliability. This may, but need not, mean that it is also required with absolute precision.
- In most problems for the sake of which computing machines are being built [...] **the precision that is wanted is quite limited**. That is the data of the problem are only given to a limited precision, and the result is only wanted to a limited precision. This is quite compatible with absolute mathematical rigor [...]

Von Neumann 1948/1961: 324-325

# Il progetto finale di Von Neumann

- I suspect that a deeper mathematical study of the nervous system [...] **will affect our understanding of the aspects of mathematics itself that are involved**. In fact, it may alter the way in which we look on mathematics and logics proper.

Von Neumann 1958:2

# Riferimenti bibliografici


- Aspray W. (1990) *John von Neumann and the Origins of Modern Computing*, Mit Press, Cambridge (Mass.).
- Heims S. (1982) *John von Neumann and Norbert Wiener: From Mathematics to the Technologies of Life and Death,* Mit Press, Cambridge (Mass.).
- Letter from Von Neumann to Wiener, 29 nov. 1946
- Von Neumann J. (1948/1961) “General and logical Theory of automata”, *Hixon Symposium*, reprinted in Taub A.H. (ed) *Collected Works*, Vol.V: 288-328.
- Von Neumann J. (1952/1956) “Probabilistic logics and the synthesis of reliable organisms from unreliable components” in C. E. Shannon, W. Ross Ashby, J. McCarthy (Eds.) Automata Studies, Princeton University Press, Cambridge (Mass.): 43-98.
- Von Neumann J. (1958) *The computer and the brain*, Yale Univ. Press, New Haven.
- Von Neumann J. (1966) *Theory of self-reproducing automata* (edited and completed by A.W. Burks), University of Illinois Press, Urbana.